Stress-Distribution Measurement by Means of Vibration Method

MASAO YAMANA*
University of Tokyo, Tokyo, Japan

An efficient method for stress-distribution measurement is summarized. The concept of measurement is somewhat similar to the concept behind the medical stethoscope. By applying a hand-held extensometer at various places on the surface of a structure subjected to a dynamic load of a finite amplitude, the distribution of the strain can be measured much more precisely and quickly than by conventional methods such as the static-load test with electric-resistance wire strain gages. This method can be applied to a small structural component as well as to a large complete airplane. Two types of hand-held extensometers are used: a mechanical type, modified from a conventional one; and an electronic type, the indicator of which shows the total amplitude of dynamic strain independently of its frequency. Several procedures for applying a suitable dynamic load to the structure, according to its size and shape, are illustrated, such as the application of fatigue testing machines and the resonance method. The methods for determining the load acting on a specified station of a structure in resonant vibration, including the additive-mass method applied to a complete airplane, are described and illustrated by examples.

1. Introduction

THE increasing importance of the fatigue strength in aircraft structures has brought the need for detailed stress-distribution measurements of the structures and their components. However, the conventional electric-resistance wire strain gage is not sufficient to determine the precise stress distribution of a structure, because of the inherent limitations in its application. Investigations of the method for stress-distribution measurement, which is free from such limitations, are summarized in the present paper.

The concept behind this method is somewhat similar to the concept behind the medical stethoscope; a hand-held extensometer is applied at various places on the surface of a structure that is subjected to a dynamic load of a finite amplitude. Typical structures under such a load are shown schematically in Fig. 1. Anywhere that the extensometer is applicable, the total amplitude of strain can be directly measured at once. The time necessary for one placing-and-reading normally is 5–10 sec, so whole maps of the detailed strain distribution can be quite speedily drawn. This method can be applied to a small structural component as well as to a large complete airplane. Thus, the present method offers much more precise measurements with an efficient procedure than do the conventional methods such as a static-loading test with electric-resistance wire strain gages. The methods for applying a dynamic load and for determining the load acting at a specified station of the structure, the concept of which is the main scope of the present paper, are described and illustrated by examples.

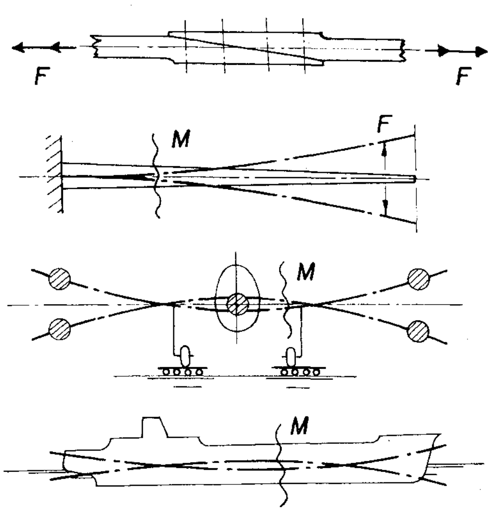

Fig. 1 Concept of method (schematic); F = force, M = bending moment.

2. Hand-Held Extensometer for Dynamic-Strain Measurement

Two types of hand-held extensometers were devised by the author and his collaborators: a mechanical one modified from a conventional Huggenberger-type extensometer,[1] and an electronic one.[2]† Figure 2 shows the electronic extensometer, consisting of a power unit and an amplifier-indicator unit. The detecting head, which is essentially a differential transformer with a set of E-I cores, is provided with a pair of knife edges, one movable and one fixed. The I core, to which the movable knife edge is fixed, is constrained by the spring action of the housing to shift only along the line through the knife edges. Figure 3 is the block diagram of the whole apparatus. The carrier wave of 5 kc/sec, generated by an oscillator, is modulated in amplitude by the

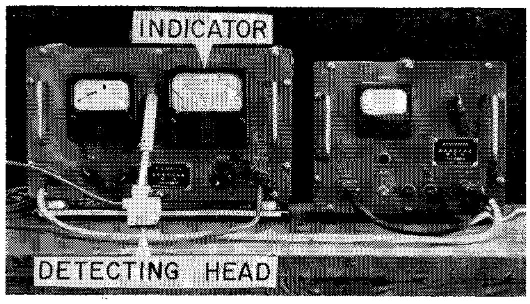

Fig. 2 Differential-transformer-type extensometer.

Presented as Preprint 65–785 at the AIAA/RAeS/JSASS Aircraft Design and Technology Meeting, Los Angeles, Calif., November 15–18, 1965; received November 29, 1965, revision received August 1, 1966.

* Professor of Aeronautics, Department of Aeronautics, Faculty of Engineering. Member AIAA.

† The electronic extensometer is available commercially at the following manufacturer: Japan Aviation Electronics Industry Ltd., Tokyo Skyline Bld., 32 Nanpeidai-machi, Shibuya-ku, Tokyo, Japan.

displacement of the movable knife edge, being transferred through the carrier-frequency amplifier to the first-detector circuit. The d.c. component of the output from the first detector is used to monitor the predetermined operating condition of the detecting head. The signal of the cyclic displacement, which is reproduced in the a.c. component of the output from the first detector, is transferred through the low-frequency amplifier to the second-detector circuit, giving a d.c. output proportional to the amplitude of the cyclic displacement in a good linear relation, except for the case of the 0.5-μ full scale of lesser proportionality. The full range of the indicator can be set by a selector switch to 15, 5, 1.5, or 0.5μ of the total amplitude of the cyclic displacement. The gage length is 20 mm in the present model, so these full ranges nearly correspond to the stresses of $\pm$(3, 1, 0.3, and 0.1) kg/mm^2, respectively, for aluminum alloy. The indication is practically independent of the frequency of the dynamic strain between 120 and 1000 cycles/min (with a deviation smaller than 1%). The mass of the moving part of the detecting head has been made light enough, and the indication is scarcely affected by the amount of the contact load from 1 to 4 kg (with a deviation smaller than 1%). The variation of the indication for the fluctuation of the a.c.-line voltage is about 0.25%/v because a voltage regulator was used.

The mechanical extensometer also is useful for a rough preliminary measurement of strain and for measuring slips between bolted or riveted members. The metal pointer of the original unit was displaced by a fine bamboo rod or a dead grass-stem, and its lever arm was provided with lightening holes to reduce inertia masses. Dynamic-strain measurement by this improved unit is possible up to about 800 cycles/min without appreciable loss of accuracy.[1]

3. Dynamic Loading by a Fatigue-Testing Machine

Experiments on small specimens are carried out favorably by using a fatigue-testing machine that supplies a regular sinusoidal dynamic load of a desired amplitude. Figure 4a shows a spar-fitting attached to the machine of Losenhausen type. The distribution of strains and that of slips between the bolted flanges can be precisely measured. Figure 4b shows the distribution of slips between the bolted flanges, measured on the same fitting as in Fig. 4a. The repeated load of $\pm$2000 kg is superposed on the tensile load of 4000 kg. The slip $S_{i,j}$ between the flanges i and j can be calculated by the relation

$$S_{i,j} = \delta_{i,j} - \tfrac{1}{2}(\delta_i + \delta_j) \tag{1}$$

where δ_i and δ_j are the displacements between two knife

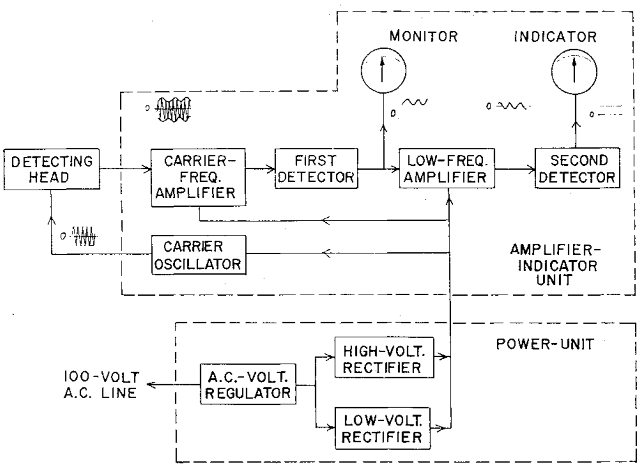

Fig. 3 Block diagram of electronic extensometer.

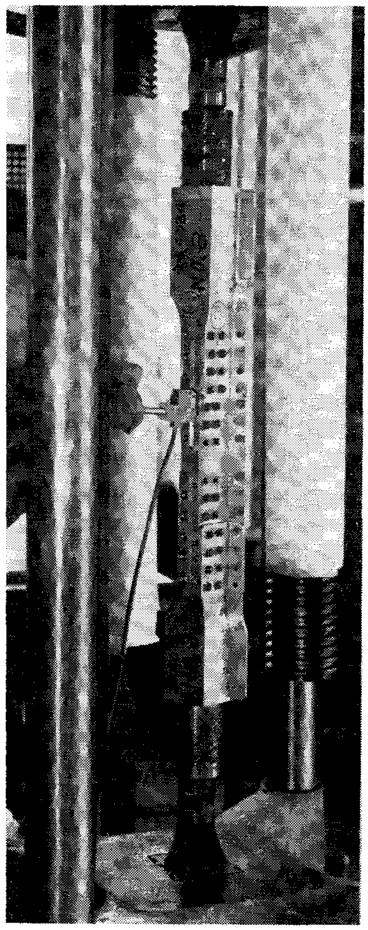

Fig. 4a Spar-fitting attached to pulsating machine.

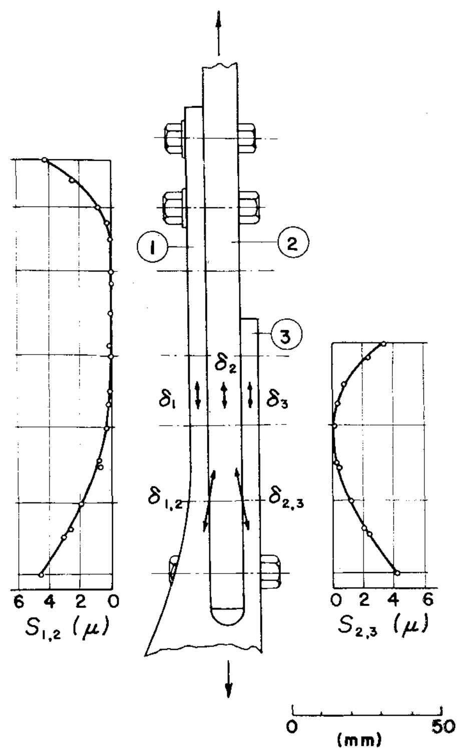

Fig. 4b Distribution of slip between bolted flanges of spar-fitting shown.

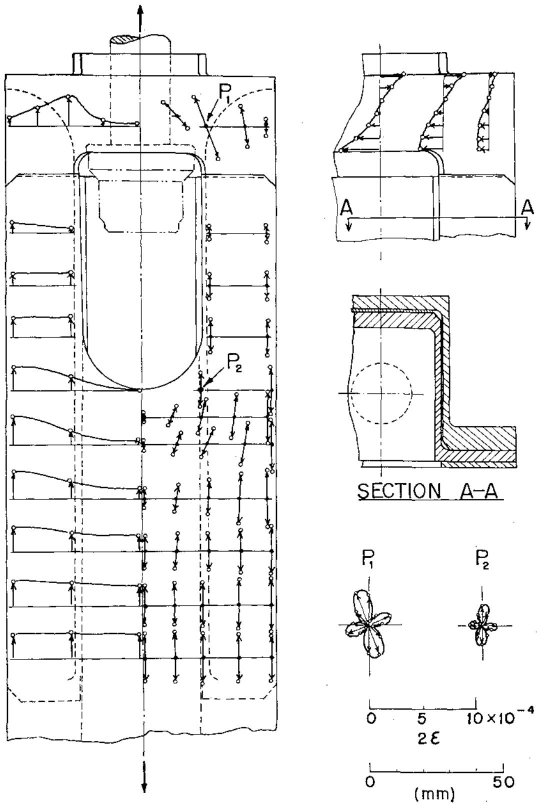

Fig. 4c Distribution of total amplitude of dynamic strain measured on spar-fitting.

edges of the extensometer placed, respectively, along the flanges i and j at a station, and $\delta_{i,j}$ is the displacement, at this same station, when the extensometer is placed so that each of the two knife edges comes into contact with the adjacent member (i, j) along the boundary. The slip at the station of each bolt can be considered roughly proportional to the shearing force acting on the bolt. Therefore, the slip distribution has a significant effect on the fatigue failure of the fitting. Figure 4c shows the dynamic strain measured on another fitting under the same load as previously described. The distribution of the vertical strain is shown in the left half and the major principal strain and the horizontal strain in the right. Polar diagrams are shown at two points P_1 and P_2, for example.

Experiments on relatively small structures also can be made by means of the same technique as the fatigue-testing machine. Figure 5a shows an example. An airplane tail is attached to a concrete column, and the repeated bending moment is applied to the tail through two cables that are attached, respectively, to each of the wooden frames fixed near wing-tips. The frequency of repeated load is made low enough, compared with the natural frequency of the structure, so that no appreciable effect of inertia forces may occur. The amplitude of working load on the wooden frame can be determined by using the deflection-load relation obtained by a static test. By applying the hand-held extensometer as shown in Fig. 5b, the strain distribution and the slips between the rivet head and the riveted plate are precisely measured.

4. Resonance Method

The resonance method is useful for large structures. When the structure is attachable to a supporting jig, the amplitude of the dynamic load (shearing force, and bending torsional moments) acting on the specified stations where the strains should be measured can be determined easily by comparing the strains on several selected spots with those of a conventional static test.

As an example, the detailed distribution of the total amplitude of dynamic spanwise strain $2\epsilon_1$ is illustrated in Fig. 6, which was measured around a section of a semimonocoque main wing under resonant bending vibration.[3] In this figure, the total amplitude of the strain is represented by the line normal to the contour of the section. The wing is attached to a vertical flat face of an iron-jig at its root section. A wooden frame is fixed near the wing-tip as an additive mass. A force is applied to the frame through a long rubber cord to generate a static moment, and an exciting force for resonant vibration is also applied to the frame through a coil spring. Both forces are exerted so as to pass through the elastic axis of the wing to prevent torsional vibration. To maintain the constancy of the amplitude in resonant state, two small water-dampers are attached to the fore and aft ends of the frame.

The strains were measured at the middle station between two adjacent ribs where the chord length was 2.577 m. To investigate the effect of spanwise stresses on the effective breadth of the skin, three different values of static moment M_0 were superposed on a dynamic moment of the amplitude M_1, as follows:

$$M = M_0 + M_1 \sin\omega t \tag{2}$$

where t is the time and ω is the circular frequency. The values of M_0 of cases a, b, and c shown in Fig. 6 are 521, −105, and −1044 m·kg, respectively, and the value of $2M_1$ is 460 m·kg throughout the cases. The positive (negative) sign of the static moment means upward (downward) bending. The determination of the total amplitude of dynamic moment at the station was performed by comparing the strains at four points, two on the upper and two on the lower surfaces along the spars, with the strains determined by static test, in which a downward or upward load was applied on the aforementioned wooden frame.

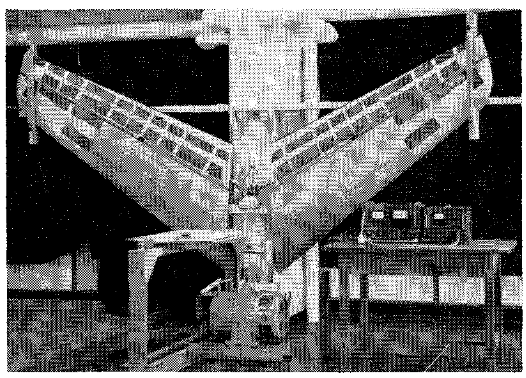

Fig. 5a Simple setup for supplying repeated load to horizontal tail.

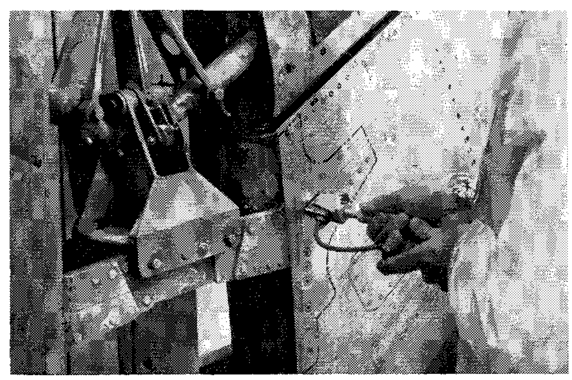

Fig. 5b Hand-held extensometer applied.

5. Resonance Method (Continued)—Complete Airplane

When static testing is adopted, the stress-distribution measurement for a complete airplane requires a fairly complicated setup: supporting jigs, loading racks, etc. The resonance method would be applied generally with much simpler procedures for the same purpose. For example, by employing an exciting equipment and an appropriate extensometer, a complete service airplane (not one allotted for a strength or fatigue test) can be examined with small stresses, so that any effects on the fatigue life of the airplane might not remain.

The additive-mass method was devised for determination of the amplitude of the load acting at the specified stations on which the strains are measured.[4] In the usual stress analysis, the load is divided into shearing force and bending and torsional moments. When secondary deformations are not appreciable, these force and moments as well as the strains can be linearly combined again. Therefore, the same treatment can be adopted also for the resonance method.

a. Determination of Bending Moment

Here the main wing of a complete airplane in a resonant vibration in its lowest mode, as shown in Fig. 7, is considered. One mass m_0 is fixed at the center and two equal masses m_1 are fixed symmetrically near the wing-tips, so as to satisfy the following relation:

$$a_0 m_0 = 2a_1 m_1 \tag{3}$$

where a_0 and a_1 are, respectively, the amplitudes of bending vibration at the stations where m_0 and m_1 are fixed. When these additive masses are sufficiently small compared with the original mass of the wing, no appreciable change will be produced in the normal function, namely, the shape of the curve of deflection. Table 1 explains the notations introduced in both cases shown in Fig. 7.

Remembering that the amplitude of inertia force due to the original wing mass, and hence M_p, is proportional to the square of the frequency, and assuming that the same curve of deflection holds for cases a) and b) for the same value of a_1, it follows that

$$M = M_p(n/n_p)^2 + M_m \tag{4}$$

where M_m is the bending moment caused by the inertia force of m_1.

Similarly, for strains,

$$\epsilon = \epsilon_p(n/n_p)^2 + \epsilon_m \tag{5}$$

where ϵ_m is the strain corresponding to M_m.

The value of ϵ_m can be calculated by putting the measured values of ϵ_p, ϵ, n_p, and n into Eq. (5). The value of M_m is readily determined by a_1, n, m_1, and the distance l between the position of m_1 and the specified station in consideration, as shown in Fig. 7. Finally, M_p and M can be determined by the relations

$$M_p = (\epsilon_p/\epsilon_m)M_m \qquad M = (\epsilon/\epsilon_m)M_m \tag{6}$$

Table 1 Notations introduced in Fig. 7

Case	Frequency, cycles/min	Amplitude of spanwise strain	Amplitude of bending moment, m·kg
a	60 n_p	ϵ_p	M_p
b	60 n	ϵ	M

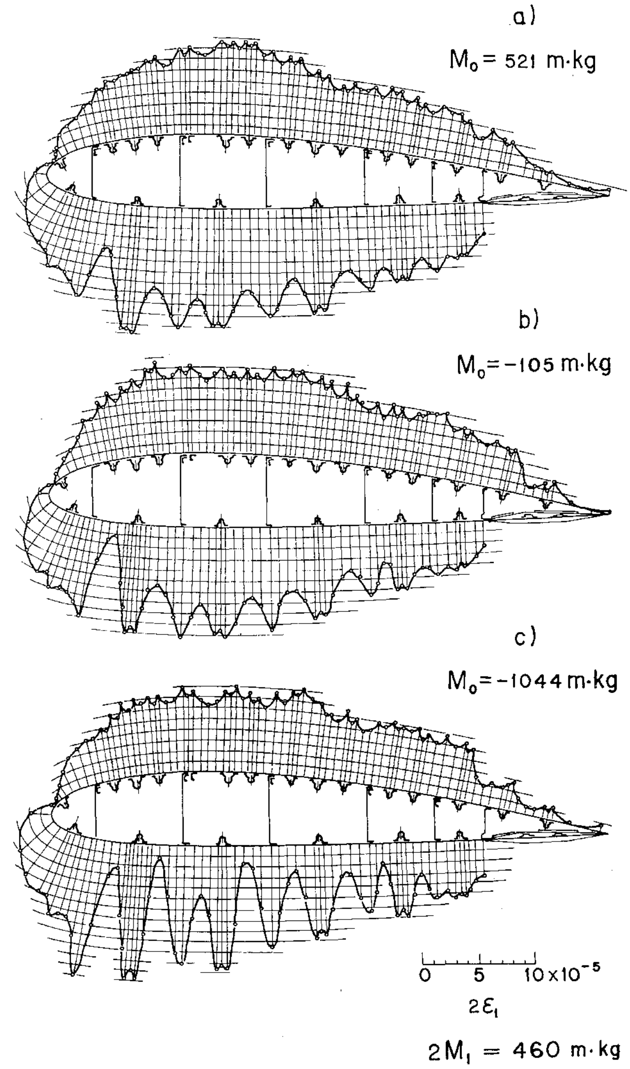

Fig. 6 Distribution of total amplitude of dynamic strain around section of wing.

If the increase in number of additive masses is desirable for minimizing the change of the curve of deflection, the distribution of masses should satisfy the relation

$$\sum_i y_i m_i = 0 \tag{7}$$

where y_i and m_i are, respectively, the deflection and the additive mass at the spanwise station i. The distribution of bending moments can be determined by a similar procedure.

As to the shearing force, much the same procedure as in the case of bending moment is applicable, replacing the

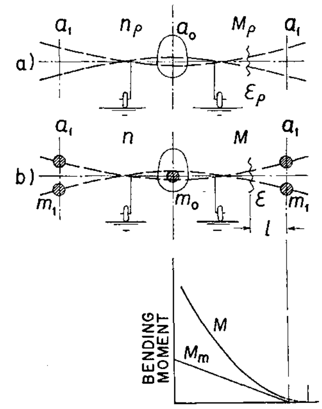

Fig. 7 Determination of amplitude of bending moment determined by additive-mass method.

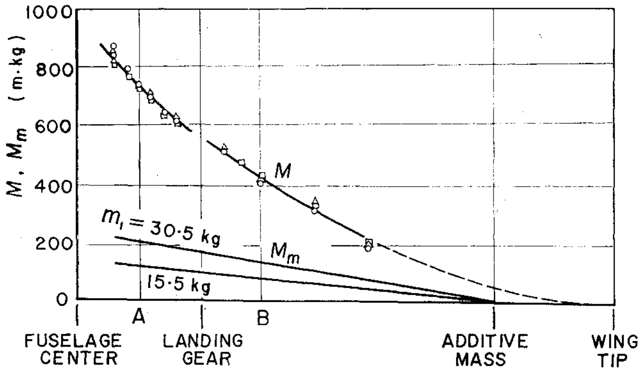

Fig. 8a Distribution of amplitude of bending moment determined by additive-mass method.

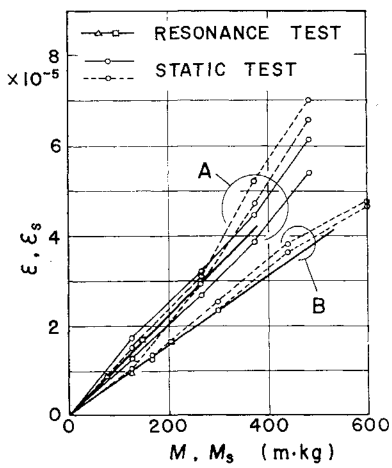

Fig. 8b Amplitude of bending moment vs amplitude of strain compared with bending moment-strain relation of static test.

bending moment and the spanwise strain by the shearing force and the corresponding strain, respectively.

b. Experiments

The validity of the present method was examined by using a single-engined monoplane, 14.36 m in span.[5] Three additive masses were used in the same way as described in the preceding paragraph. Excitation was exerted at the station of a wing-tip mass. Four spots at one station were chosen for the strain measurement, at the front and rear spars on the upper and lower surfaces. Such sets of measurement were made at seven stations along the span.

Figure 8a shows the distribution of bending moments obtained by the foregoing procedure. The relations between the bending moments and strains are compared with those obtained by a static test. In this static test, the main wing was supported at the roots and a downward load was applied at each wing-tip. The bending moment-strain relations at two stations A and B shown in Fig. 8a are illustrated in Fig. 8b. In this figure, M and M_s denote the amplitudes of dynamic bending moment and static bending moment, respectively, and ϵ and ϵ_s are the corresponding strains. The good coincidence between the results from both tests indicates that three additive masses give a satisfactory result for determining the bending moment by the present method.

An example of the strain distribution measured on a different airplane is illustrated in Fig. 9. The total amplitude of spanwise strain 2ϵ, measured at several stations on the upper and lower surfaces of the wing, and the corresponding total amplitude of bending moment $2M$, are shown in the figure. The airplane was tested immediately before going into service, and measurement was carried out in about 5 hr. This was a single-engined monoplane with a nose-wheel. During the experiment, the airplane was laid down with its three wheels on the platform in the usual way. Similar experiments were conducted on several airplanes. For an airplane with fairly long landing gears, roller bearings were used between the platform and wheel-tires to avoid creating the fixing moment generated by the friction between them.

c. Determination of Torsional Moment

It is much easier to determine torsional moment than bending moment. The procedure shown in Fig. 10 gives the determination of the apparent moment of inertia I_0, which includes all contributions of the oscillating parts except the additive masses. Therefore, the effective moment of inertia I is expressed by

$$I = I_0 + 2ml^2 \qquad (8)$$

where l denotes the distance of an additive mass m from the center axis of torsional vibration. The total amplitude of torsional moment $2Q$ is calculated as

$$2Q = 2I(a/l)(2\pi n)^2 \qquad (9)$$

where a is the amplitude of an additive mass and n is the frequency in cps. Stress-distribution measurements for several airplane fuselages in torsional resonant vibration were carried out[4,6] by the aforementioned procedure, in which the additive masses were attached to horizontal tails.

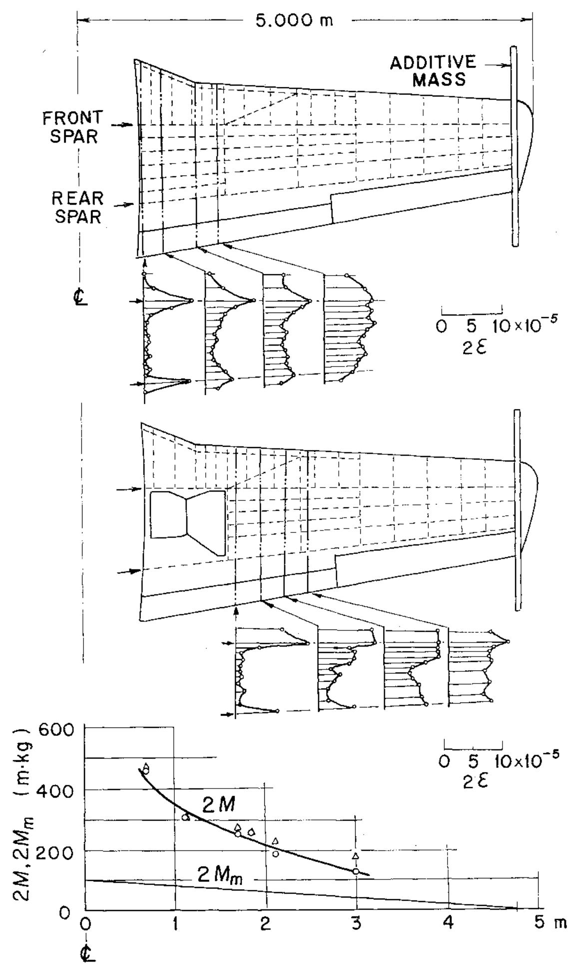

Fig. 9 Distribution of total amplitudes of dynamic strain and bending moment obtained by resonance method on complete airplane.

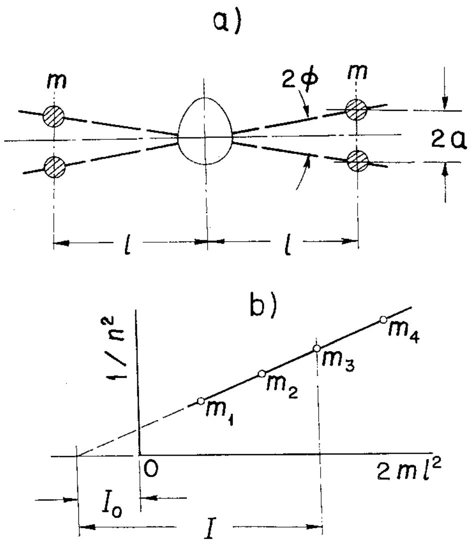

Fig. 10 Determination of torsional moment by additive-mass method.

6. Historical Review

Application of the present method is gradually increasing in Japan. In other countries, most of the previous tests on airplane structures using the vibration technique seem to have been carried out for the purpose of examination of flutter characteristics or for speedy determination of fatigue life, but not for the purpose of quick and detailed stress-distribution measurement.

The first paper on the present method appeared in 1938.[4] Three papers followed in the same year.[1,3,5] At that time, strains were measured by a mechanical extensometer that required considerable skill in measurement. The electronic extensometer was reported in 1961,[2] and several kinds of tests have been carried out thereby. The procedure of this method was summarized in a published paper,[6] and the applicability of this method to the stress-distribution measurement of ships was presented in 1964.[7]

7. Conclusions

1) The stress distribution on the surface of a structure can be surveyed quickly and in detail by the present method. For a structure on which a high degree of analytical prediction is difficult, this method is especially useful.

2) This method can be used throughout the range of sizes from a small structural component to a complete airplane.

3) By the resonance method with additive masses, a complete service airplane as well as one allotted for a strength or fatigue test can be examined with such small stresses as have no appreciable effect on its fatigue life.

4) This method is useful for indicating the stress-flow precisely before the usual fatigue testing, which indicates only the weakest regions.

5) This method will be promising as a technique for a non-destructive examination of the strength and fatigue life of a very large airplane for which the usual destructive testing is too expensive and laborious.

6) The method offers a structural designer a useful means for checking his design and refining his design feeling.

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